Dept. of Electrical Engineering
Faculty of Engineering
Assiut University
1st Semester - Final Exam
2014/2015 - January 2015

Course: Numerical Analysis
Code: E226
2nd year
Time: 3 Hours
Marks: 100

Important remarks
- This exam measures ILOs no.: a.1.1, a.1.2, a.5.1, a.5.2, a.18.1, b.1.1, b.2.1, b.3.1, b.11.1, c.5.1, c.7.1
- No. of questions: 4
- No. of pages: 2
- **Round your answers to four digits after the decimal point**

Answer the Following Questions

Question no. 1 (27 points)

A. Let $f(x) = 3\sin^2(\pi x/6)$. Construct the divided-difference table based on the nodes $x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 3, and\ x_4 = 4$.

Find the *Newton polynomial* $P_3(x)$, and evaluate this polynomial at $x = 1.5$. **(14 points)**

B. Write a Matlab function to solve the **lower-triangular** system **AX = B** by the method of ***forward substitution***. Name the function **forsub**. **(7 points)**

C. Derive the general formula of the ***secant method***. **(6 points)**

Question no. 2 (29 points)

A. Given a set of data,

x_k	-1	0	1	2	3
y_k	3.08	4.44	6.19	8.25	10.48

(i) Find the least-squares curve $f(x) = L/(1 + Ce^{Ax})$, with $L = 20$ **(15 points)**

(ii) Apply the ***O(h²)*** **centered-difference** formula of the derivative to find $f''(0)$. **(3 points)**

B. Write a Matlab function to approximate a root of $f(x) = 0$ using the ***accelerated Newton-Raphson method***. **(11 points)**

Question no. 3 (25 points)

A. Use the ***recursive Simpson rule*** to compute the approximation S(2) for the integral $\int_1^5 dx/x$. Compute the relative error in this case. **(12 points)**

B. Write a Matlab function to approximate the integral $\int_a^b f(x)$ using the ***recursive Trapezoidal rule***. Name the function <u>rctrap</u>. **(13 points)**

Question no. 4 (19 points)

A. Find the triangular factorization $A = LU$ for the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 0 & 4 & 3 \\ 4 & 2 & 2 & 1 \\ -3 & 1 & 3 & 2 \end{bmatrix}$$

(9 points)

B. Let $f(x)$ be a polynomial of degree $\leq N$. Let $P_N(x)$ be the ***Lagrange polynomial*** of degree $\leq N$ based on the $N+1$ nodes $x_0, x_1, \ldots, x_N$. Show that $f(x) = P_N(x)$ for all x.

(4 points)

C. Use the ***Lagrange polynomial*** to derive the ***$O(h^2)$*** **forward-difference formula** for $f'(x)$.

(6 points)

Best Wishes

Dr. Noha Medhat