E222 CIRCUIT THEORY
First Term Examination, 2014/2015

## Model Answer

## Question \#1: (12 Points)

The circuit shown in Fig. 1 is used to measure the change in resistance experienced by strain gages.
a) Derive an expression for the output voltage $V_{\text {out }}$ in terms of the resistance values and the reference voltage $\boldsymbol{V}_{\text {ref }}$, assuming ideal op-amp and neglecting $\Delta \boldsymbol{R}^{2}$ w.r.t. $\boldsymbol{R}^{2}$.
(8 Points)
b) If $\boldsymbol{R}=160 \Omega, \Delta \boldsymbol{R}=1 \Omega, \boldsymbol{R}_{\boldsymbol{f}}=1.2 \mathrm{~K} \Omega$, and $\boldsymbol{V}_{\text {ref }}$ $=8 \mathrm{~V}$; find the value $\boldsymbol{V}_{\text {out }}$.
(4 Points)

Fig. 1


Expression for $V_{\text {out }}$ :

$$
V_{o u t}=\frac{2 R_{f} \Delta R}{R^{2}} \cdot V_{r e f}
$$

Value of $V_{\text {out }}$ :

$$
V_{\text {out }}=0.75 \mathrm{~V}
$$

Question \#2: (16 Points)
In the circuit shown in Fig.2, the initial currents in inductors $\boldsymbol{L}_{\mathbf{1}}$ and $\boldsymbol{L}_{2}$ are 8A and 1A respectively. The switch is opened at $t=0$.
a) Find $i(t)$, for $t \geq 0$.
b) Find $v(t)$, for $t \geq 0$.
c) Find $\boldsymbol{i}_{\mathbf{1}}(\boldsymbol{t})$ and $\boldsymbol{i}_{\mathbf{2}}(\boldsymbol{t})$, for $\boldsymbol{t} \geq 0$.
(8 Points)
d) Determine the total energy stored in the inductors as $t \rightarrow \infty$.
(2 Points)


Fig. 2
d) Determine the total energy stored in the
(2 Points)
(4 Points)

$$
i\left(0^{+}\right)=9 \mathrm{~A}
$$

$$
i(\infty)=0
$$

$$
\tau=0.5 \mathrm{~S}
$$

$$
i(t)=9 e^{-2 t} A
$$

$$
v(t)=36 e^{-2 t} V
$$

$$
i_{1}(t)=-2-6 e^{-2 t} \mathrm{~A}
$$

$$
i_{2}(t)=2-3 e^{-2 t} A
$$

## Energy stored in the inductors as $t \rightarrow \infty=18 \mathrm{~J}$

## Question \#3: (12 Points)

The switch in the circuit shown in Fig. 3 has been in position (a) for a long time. At $t=0$, it moves to position (b). Find $\boldsymbol{i}\left(\mathbf{0}^{+}\right), \boldsymbol{v}_{\mathrm{c}}\left(\mathbf{0}^{+}\right)$, $\boldsymbol{d i}\left(\mathbf{0}^{+}\right) / \boldsymbol{d t}$, the roots of the characteristic equation $s_{1}, s_{2}$ and $i(t)$ for $t \geq \mathbf{0}$.


Fig. 3

$$
v_{C}\left(0^{+}\right)=50 \mathrm{~V}
$$

$$
s_{2}=-8000-j 6000
$$

$$
i(t)=1.6736 e^{-8000 t} \sin (6000 t) A
$$

## Question \#4: (12 Points)

A three-phase $\Delta$-connected -ve sequence source having the phase voltage $V_{a b}=240 \angle 0^{\circ} \mathrm{V}$ and negligible source resistance. The source supplies a resistive unbalanced $\Delta$-connected load having impedances: $\boldsymbol{R}_{A B}=60 \Omega, \boldsymbol{R}_{B C}=40 \Omega$, and $\boldsymbol{R}_{C A}=$ $80 \Omega$. The three lines connecting the source to the load have negligible resistances. The load power is measured using the two wattmeter method. The first wattmeter $W_{1}$ is connected between lines A and B, while the second one $W_{2}$ is connected between lines C and B . Find the following:

| The phase current $\overline{\boldsymbol{I}_{A B}}$ at the load $=$ | $4 \angle 0^{\circ} \mathrm{A}$ |
| :---: | :---: |
| The Line current $\overline{\boldsymbol{I}_{\boldsymbol{A}}}=$ | $6.08 \angle 25.3^{\circ} \mathrm{A}$ |
| The Line current $\overline{\boldsymbol{I}_{\boldsymbol{C}}}=$ | 7.94 $\angle 79.1^{\circ} \mathrm{A}$ |
| The reading of $W_{1}=$ | 1.32 KW |
| The reading of $W_{2}=$ | 1.8 KW |
| The total power dissipated in the load $=$ | 3.12 KW |

## Question \#5: ( $\mathbf{1 4}$ Points)

The voltage source $v_{g}$ drives the circuit shown in Fig.5. The response signal is the voltage across the capacitor, $v_{o}$.
a) Calculate the numerical expression for the voltage transfer function $\boldsymbol{H}_{v}(\boldsymbol{s})=\boldsymbol{V}_{\boldsymbol{o}}(\mathbf{s}) / \boldsymbol{V}_{g}(\mathbf{s})$. (6 points)
b) Calculate the numerical values for the poles and zeros of the transfer function. (4 points)
c) The circuit is driven by a step voltage source, namely, $v_{g}=\mathbf{5 0 u}(t)$, find $v_{0}(t)$. (4 points)


Fig. 5
$H_{v}(s)=\frac{1000(s+5000)}{s^{2}+6000 s+25 \times 10^{6}}$
Poles:
$-3000+j 4000,-3000-j 4000$
Zeros:
-5000, $\infty$
$V_{\text {out }}(t)=\left[10+11.18 e^{-3000 t} \cos \left(4000 t-153.4^{\circ}\right)\right] u(t) V$

## Question \#6: (12 Points)

The ideal transformer used in the circuit of Fig. 6 has a turns ratio $N_{2} / N_{1}=3$.
a) Find the reflected impedance at terminals a-b of that transformer.
(6 Points)


Fig. 6
b) Calculate the value of the currents $\boldsymbol{I}_{\mathbf{1}}, \boldsymbol{I}_{\mathbf{2}}, \boldsymbol{I}_{\mathbf{3}}$, and $\boldsymbol{I}_{\mathbf{4}}$. (6 Points)

$$
Z_{R}=\left(\frac{a}{1+a}\right)^{2} R_{L}=54 \Omega
$$

$$
I_{1}=0.584 \angle 30.1^{\circ} \mathrm{A}
$$

$$
I_{2}=0.195 \angle 30.1^{\circ} \mathrm{A}
$$

$$
I_{3}=0.779 \angle 30.1^{\circ} \mathrm{A}
$$

$$
I_{4}=4.35 \angle-33.3^{\circ} \mathrm{A}
$$

## Question \#7: (6 Points)

a) Calculate the impedance of the circuit shown in Fig. 7 at radian frequency of $2 \mathrm{Krad} / \mathrm{S}$. (2 Points)
b) At what finite frequency ( $\omega_{r}$ ) does the impedance of the circuit become purely resistive? What is the impedance at that frequency?
(4 Points)


Fig. 7

$$
Z(2 \mathrm{Krad})=9-j 12 \Omega
$$

$$
\omega_{r}=4 \mathrm{Krad} / \mathrm{S}
$$

$$
Z\left(\omega_{r}\right)=15 \Omega
$$

## Question \#8: (8 Points)

Sketch the Bode Diagram of the voltage transfer function: $H_{v}(s)=\frac{1000(s+100)}{(s+10)(s+1000)}$


## Question \#9: (8 Points)

The y parameters for the two-port network in Fig. 9 are: $y_{11}=2 \mathrm{mS}, y_{12}=-0.2 \mathrm{mS}, y_{21}=10 \mathrm{mS}$, and $y_{22}=-0.5 \mathrm{mS}$. Find $\boldsymbol{V}_{\mathbf{1}}, \boldsymbol{V}_{2}, \boldsymbol{I}_{1}$, and $\boldsymbol{I}_{\mathbf{2}}$.


Fig. 9

$$
V_{2}=-40 \mathrm{~V}
$$

$$
V_{1}=-1.6 \mathrm{~V}
$$

$$
I_{1}=4.8 \mathrm{~mA}
$$

$$
I_{2}=4 \mathrm{~mA}
$$

$$
I_{2}=4 \mathrm{~mA}
$$

$$
1-2
$$

